Math 304 Midterm 1 Sample

Name: ____________________

This exam has 9 questions, for a total of 100 points.

Please answer each question in the space provided. You need to write **full solutions**. Answers without justification will not be graded. Cross out anything the grader should ignore and circle or box the final answer.

Question	Points	Score
1	18	
2	10	
3	10	
4	10	
5	10	
6	10	
7	16	
8	10	
9	6	
Total:	100	

Question 1. (18 pts)

Determine whether each of the following statements is true or false. You do NOT need to explain.

(a) Let V be a linear subspace of $\mathbb{R}^n$. We have vectors $v_1, \cdots, v_k$ and $w_1, \cdots, w_\ell$ in V. Suppose $v_1, \cdots, v_k$ are linearly independent, and $w_1, \cdots, w_\ell$ span V. Then $k \leq \ell$.

(b) Let v and w be two nonzero vectors in $\mathbb{R}^4$. Then v and w are linearly independent if only if v is not a scalar multiple of w.

(c) An $(n \times n)$ matrix is nonsingular if and only if it is row equivalent to the $(n \times n)$ identity matrix I_n.

(d) The following matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

is a row echelon form.

(e) Let A, B, C be three $(n \times n)$ square matrices. If $AB = AC$, then $B = C$.

(f) The linear system

$$\begin{aligned} x &+ 10y - 3z = 3 \\ 3x &+ 4y + 9z = 1 \\ 2x &+ 5y - 2z = 8 \end{aligned}$$

has exactly two solutions.

Question 2. (10 pts)

Solve the following linear system

$$\begin{cases} 2x_1 + 2x_2 - 3x_3 + x_4 + 13x_5 = 0 \\ x_1 + x_2 + x_3 + x_4 - x_5 = 0 \\ 3x_1 + 3x_2 - 5x_3 + 14x_5 = 0 \\ 6x_1 + 6x_2 - 2x_3 + 4x_4 + 16x_5 = 0 \end{cases}$$

Question 3. (10 pts)

Compute the determinant of the following matrix

$$\begin{pmatrix} 2 & 1 & 2 & 1 \\ 3 & 0 & 1 & 1 \\ -1 & 2 & -2 & 1 \\ -3 & 2 & 3 & 1 \end{pmatrix}$$

Question 4. (10 pts)

Determine whether the following matrix is nonsingular. If yes, find its inverse.

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 3 & 7 & 7 & 4 \\ 2 & 4 & 5 & 1 \\ 2 & 4 & 4 & 1 \end{pmatrix}$$

Question 5. (10 pts)

Determine whether the following are subspaces.

(a) Let $\mathbb{P}_2$ be the vector space of all polynomials with degree equal to or less than 2. Is

$$W = \{p \in \mathbb{P}_2 \mid p(1) = 0\}$$

a subspace of $\mathbb{P}_2$?

(b) Is $S = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$ a subspace of $\mathbb{R}^3$?

Question 6. (10 pts)

Determine whether $x = \begin{bmatrix} 4 \\ 5 \\ 6 \\ -1 \end{bmatrix}$ lies in the linear span of the vectors

$$v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 4 \\ -1 \\ 2 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 3 \end{bmatrix}.$$

Question 7. (16 pts)

Determine whether

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

form a basis of $\mathbb{R}^3$.

Question 8. (10 pts)

Let $\mathbb{P}_2$ be the vector space of all polynomials of degree equal to or less than 2. Determine whether the following polynomials in $\mathbb{P}_2$

$$p_1(t) = t - 1$$

$$p_2(t) = t + 1$$

$$p_3(t) = (t - 1)^2$$

are linearly independent or not.

Question 9. (6 pts)

Suppose A and B are nonsingular $(n \times n)$ matrices. We know that AB is also nonsingular. Show that

$$(AB)^{-1} = B^{-1}A^{-1}.$$